**When evaluating limits, we are checking around the point that we are approaching, NOT at the point.**

**Every time we find a limit, we need to check from the left and the right hand side**
(Only if there is a BREAK at that point).

**1-2 Finding Limits Graphically and Numerically**

<table>
<thead>
<tr>
<th>No breaking point</th>
<th>Hole in the graph</th>
<th>piece-wise function</th>
<th>radicals</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
</tbody>
</table>

\[
\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a} f(x) =
\]

**Asymptotes**

\[
\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a} f(x) =
\]

**If left and right hand limits DISAGREE, then the limit Does Not Exist (DNE) at that point.**

**If left and right hand limits AGREE, then the limit exists at that point as that value.**

**Even if you can plug in the value, the limit might not exist at that point. It might not exist from the left or right side or the two sides will not agree.**

\[
\lim_{x \to a^+} f(x) = \text{right hand limit} \\
\lim_{x \to a^-} f(x) = \text{left hand limit}
\]
**Breaking Points are points on the graph that are undefined or where the graph is split into pieces.**

**Breaking Points:**
1) Holes (when the numerator and denominator equals 0)
2) Radicals (when the radical equals 0)
3) Asymptotes (when the denominator equals 0)
4) Piece-wise functions (the # where the graph is split)

**Note:** In general when doing limits, \[ \frac{#}{0} = \infty \quad \frac{\#}{\#} = \infty \quad \frac{\#}{\#} = 0 \]

**In limits, if the two sides of a graph don’t agree, then the limit does not exist.**

1-3 Analyzing Limits Analytically

**LIMITS AT NON-BREAKING POINTS** (Very easy. Just plug in the #)

EX#1: \[ \lim_{x \to 1} x^2 = \]  
EX#2: \[ \lim_{x \to 5} \sqrt{x + 4} = \]  
EX#3: \[ \lim_{x \to 1} \frac{x - 1}{x + 1} = \]

**HOLES IN THE GRAPH** (Factor and cancel or multiply by the conjugate and cancel, then plug in #)

EX#1: \[ \lim_{x \to 3} \frac{x^2 + 3x - 18}{x - 3} = \]  
EX#3: \[ \lim_{x \to 4} \frac{\sqrt{x + 5} - 1}{x + 4} = \]

EX#2: \[ \lim_{x \to 4} \frac{x - 4}{x - 4} = \]

**TRIG. FUNCTIONS**  
You can use \[ L^1 H \text{ Rule} \] for indeterminate form

Trig. Identities to know: \[ \sin 2x = 2 \sin x \cos x \quad \cos 2x = \cos^2 x - \sin^2 x \]

**FACTS:**  
\[ \lim_{x \to 0} \frac{\sin x}{x} = 1 \quad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0 \quad \lim_{x \to 0} \frac{\tan x}{x} = 1 \]

\[ \lim_{x \to 0} \frac{\sin ax}{bx} = \frac{a}{b} \quad \lim_{x \to 0} \frac{1 - \cos ax}{bx} = 0 \quad \lim_{x \to 0} \frac{\tan ax}{bx} = \frac{a}{b} \]

EX#1: \[ \lim_{x \to 0} \frac{\sin 2x}{5x} = \]  
EX#4: \[ \lim_{x \to 0} \frac{\sin x \tan x}{x^2} = \]

EX#2: \[ \lim_{x \to 0} \frac{2 \tan 5x}{9x} = \]  
EX#5: \[ \lim_{x \to 0} \frac{8 \sin x \cos x}{3x} = \]

EX#3: \[ \lim_{x \to \frac{\pi}{2}} \frac{4 \sin 3x}{x} = \]  
EX#6: \[ \lim_{x \to 0} \frac{7 \sin^2 x \tan^2 x}{x^4} = \]
1-4 Continuity and One-Sided Limits

RADICALS  (You must first check that the limit exists on the side(s) you are checking)
If a # makes a radical negative, the limit will not exist at that #.
When we check at the breaking point (the # that makes the radical zero) there are two possible answers:
1) 0 if the limit works from the side that you are checking.
2) DNE if the limit does not work from the side that you are checking.

EX #1: \( \lim_{x \to 5^-} \sqrt{x-5} = \)  
EX #2: \( \lim_{x \to 5^+} \sqrt{x-5} = \)  
EX #3: \( \lim_{x \to 5} \sqrt{x-5} = \)

EX #4: \( \lim_{x \to 2} \sqrt{16-x^2} = \)  
EX #5: \( \lim_{x \to 5} \sqrt{16-x^2} = \)  
EX #6: \( \lim_{x \to 4} \sqrt{16-x^2} = \)

PIECE-WISE FUNCTIONS

\[ f(x) = \begin{cases} 
3 - x & \text{if } x < -3 \\
2x + 12 & \text{if } -3 \leq x < 4 \\
9 & \text{if } x \geq 4 
\end{cases} \]

The breaking points are -3 and 4.

EX #1: \( \lim_{x \to -3^-} f(x) = \)  
EX #2: \( \lim_{x \to -3^+} f(x) = \)  
EX #3: \( \lim_{x \to -3} f(x) = \)

EX #4: \( \lim_{x \to 4^-} f(x) = \)  
EX #5: \( \lim_{x \to 3} f(x) = \)  
EX #6: \( \lim_{x \to 4} f(x) = \)

EX #7: \( \lim_{x \to 7^-} f(x) = \)  
EX #8: \( \lim_{x \to -5} f(x) = \)  
EX #9: \( \lim_{x \to 2} f(x) = \)

CONTINUITY

Continuous functions have no breaks in them.
Discontinuous functions have breaks in them (Asymptotes or Holes / Open Circles).

** To check for continuity at "a", you must check left hand limits \( \lim_{x \to a^-} f(x) \) and right hand limits \( \lim_{x \to a^+} f(x) \)
as well as the value of the function at that point \( f(a) \). If all three are equal then the function is continuous at \( a \).

If \( f(a) = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) \) then the function is continuous at \( a \).
If \( f(a) \) is not equal to either one-sided limit, then the function is not continuous (discontinuous) at \( a \).
1-5 Infinite Limits

ASYMPTOTES (Since the point DNE we have to check a point that is close on the side we are approaching)

There are three possible answers when checking near the breaking point (the # that makes bottom = zero)

1) \( \infty \rightarrow \) If we get a positive answer the limit approaches \( \infty \)
2) \( -\infty \rightarrow \) If we get a negative answer the limit approaches \( -\infty \)
3) DNE \( \rightarrow \) If we get a positive answer on one side and a negative answer on the other side, then the limit DNE

EX #1: \( \lim_{x \to 7} \frac{1}{x - 7} = \)

EX #2: \( \lim_{x \to 7} \frac{1}{x - 7} = \)

EX #3: \( \lim_{x \to 7} \frac{1}{x - 7} = \)

EX #4: \( \lim_{x \to 2} \frac{3}{(x - 2)^2} = \)

EX #5: \( \lim_{x \to 0^-} \frac{\cos x}{x} = \)

EX #6: \( \lim_{x \to \pi/2^+} \tan x = \)

3-5 Limits at Infinity

LIMITS THAT APPROACH INFINITY

Check the powers of the numerator and denominator.

1) If the denominator (bottom) is a bigger power the limit = 0.
2) If the numerator (top) is a bigger power the limit = \( \infty \) or \( -\infty \).
3) If powers are the same the limit = \( \frac{\text{coefficient of the highest power of numerator}}{\text{coefficient of the highest power of denominator}} \)

EX#1: \( \lim_{x \to \infty} \frac{2x^2 + 3}{5x^2 - 7} = \)

EX#2: \( \lim_{x \to \infty} \frac{6x}{3x^2 + 1} = \)

EX#3: \( \lim_{x \to \infty} \frac{x^2}{3x - 1} = \)

EX#4: \( \lim_{x \to \infty} \frac{8 - x}{x - 8} = \)

EX#5: \( \lim_{x \to \infty} \frac{8x^2 + 5}{4 - 3x^2} = \)

EX#6: \( \lim_{x \to \infty} \frac{7x + 2}{8x^2 - 1} = \)

EX#7: \( \lim_{x \to \infty} \frac{6x^3}{2x^2 - 1} = \)

EX#8: \( \lim_{x \to \infty} 3 = \)

FINDING VERTICAL ASYMMPTOTES AND HOLES

A vertical asymptote is the # that makes only the denominator = 0.

A hole occurs at the points that make the numerator and denominator = 0 at the same time.

EX#1:

a) \( f(x) = \frac{x - 2}{x + 3} \)  b) \( f(x) = \frac{8x}{x^2 + 9} \)  c) \( f(x) = \frac{(x + 5)(x - 7)}{(x - 7)} \)  d) \( f(x) = \frac{(x + 1)(x + 4)}{(x + 4)(x - 6)} \)

vert.asym. hole vert.asym. hole vert.asym. hole vert.asym. holes

6
CH. 2 DIFFERENTIATION

2-1 The Derivative by Definition and the Tangent Line Problem

Derivative at all points

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

Derivative at the point \((a, f(a))\)

\[ f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \]

Line 1 is a secant line

slope of secant line 1 = \( \frac{f(x+h) - f(x)}{h} \)

Line 1 is a tangent line

\[ \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

means that the distance \(h\) is approaching 0 and the points get closer to each other and the two points become the same point and line 1 is now a tangent line.

The derivative of a function finds the slope of the tangent line!

EX #1: \( f(x) = x^2 - 3x + 2 \) Find \( f'(x) \) and \( f'(4) \).

Use \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)

\[ f'(x) = \]

\[ f'(4) = \]

EX #2: \( f(x) = x^2 - 3x + 2 \) Find \( f'(4) \)

Use \( f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \)

\[ f'(4) = \]
The derivative finds the slope of the tangent line.

The normal line is perpendicular to the tangent line.

**EX #3:** \( f(x) = \frac{1}{x} \)  
Find equation of the tangent line and normal line at \( x = 3 \).

**Equation of the tangent line:**

**Equation of the normal line:**
2-2 Basic Differentiation Rules, Notation and Rates of Change

Properties of Derivatives

Derivative is a rate of change; it finds the change in \( y \) over the change in \( x \), \( \frac{dy}{dx} \), which is slope.

**1st derivative** \( \Rightarrow \) max. and min., increasing and decreasing, slope of the tangent line to the curve, and velocity.

**2nd derivative** \( \Rightarrow \) inflection points, concavity, and acceleration.

<table>
<thead>
<tr>
<th><strong>Power Rule</strong></th>
<th><strong>Constant Rule</strong></th>
<th><strong>Derivative notation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^n )</td>
<td>( f(x) = c )</td>
<td><strong>Newton Notation</strong></td>
</tr>
<tr>
<td>( f'(x) = nx^{n-1} )</td>
<td>( f'(x) = 0 )</td>
<td></td>
</tr>
<tr>
<td><strong>EX:</strong> ( y = 4x^3 )</td>
<td>( y = 8 )</td>
<td><strong>Leibniz Notation</strong></td>
</tr>
<tr>
<td>( y' = 12x^2 )</td>
<td>( y' = 0 )</td>
<td></td>
</tr>
<tr>
<td><strong>EX#1:</strong> ( y = 15x^4 ) Find ( \frac{dy}{dx} ) and ( \frac{dy}{dx} \mid_{x=2} ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y'' = 6 )</td>
<td>( y'' = 6 )</td>
<td></td>
</tr>
<tr>
<td>( y'' = 6 )</td>
<td>( y'' = 6 )</td>
<td></td>
</tr>
<tr>
<td>( y'' = 6 )</td>
<td>( y'' = 6 )</td>
<td></td>
</tr>
<tr>
<td>( y'' = 6 )</td>
<td>( y'' = 6 )</td>
<td></td>
</tr>
<tr>
<td><strong>EX#2:</strong> Find ( f'(x) ) for each.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a) \quad f(x) = 500 )</td>
<td>( b) \quad f(x) = \frac{8}{x} )</td>
<td></td>
</tr>
<tr>
<td>( c) \quad f(x) = \frac{1}{x^2} )</td>
<td>( d) \quad f(x) = \frac{x}{7} )</td>
<td></td>
</tr>
</tbody>
</table>

Slope of the tangent line to the curve

**EX:** Given \( f(x) = 3x^2 - 10x \) Find equation of the tangent line and normal line at \( x = 4 \).

\( f(x) = 3x^2 - 10x \) \quad \( f'(x) = 6x - 10 \)

\( f(4) = 8 \) \quad \( f'(4) = 14 \)

**Equation of a Line (point-slope form):** \( y - y_1 = m(x - x_1) \)

**Equation of the tangent line:** \( y - 8 = 14(x - 4) \)

**Equation of the normal line:** \( y - 8 = -\frac{1}{14}(x - 4) \)
EX #3: Find the slope, write the equation of the tangent line and the equation of the normal line at $x = 3$.

$f(x) = 5x^2 - 7$

| Equation of the tangent line | Equation of the normal line |

*Trig. Functions*

2 STEPS: Derivative of the trig. function \cdot Derivative of the angle

<table>
<thead>
<tr>
<th>Function</th>
<th>Derivative</th>
<th>Function</th>
<th>Derivative</th>
<th>Function</th>
<th>Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>\sin x</td>
<td>\cos x</td>
<td>\tan x</td>
<td>\sec^2 x</td>
<td>\sec x</td>
<td>\sec x \tan x</td>
</tr>
<tr>
<td>\cos x</td>
<td>-sin x</td>
<td>\cot x</td>
<td>-\csc^2 x</td>
<td>\csc x</td>
<td>-\csc x \cot x</td>
</tr>
</tbody>
</table>

EX #4: Find each derivative

a) $y = \sin(8x)$  
b) $y = \cos(x^4)$  
c) $y = \tan(7x^3)$

d) $y = \sec(6x^2 + 5)$  
e) $y = \csc(2x^5)$  
f) $y = \cot(9x - 1)$

g) $y = \cos(\sin x)$  
h) $y = \sec(\tan 3x)$

Use $s(t) = -16t^2 + v_0t + s_0$.  
$s(t)$ = ending height  
$s_0$ = initial height  
v_0 = initial velocity  
t = time

EX #5: To estimate the height of a building, a weight is dropped from the top of the building into a pool at ground level. How high is the building if the splash is seen 2 seconds after the weight is dropped?

EX #6: A red ball is thrown upward from a building 100 feet above the ground with an initial velocity of 10ft/s. At the same time, a blue ball is thrown downward from a height of 150 feet with an initial velocity of 10ft/s. Which ball hits the ground first? How much faster?
2-3 Product and Quotient Rule

*Product Rule

4 STEPS:  Derivative of First equation · Second equation + Derivative of Second equation · First equation

\[ y = f(x) \cdot g(x) \quad \frac{dy}{dx} = f'(x)g(x) + g'(x)f(x) \]

*Quotient Rule

5 STEPS:  \( \frac{\text{Derivative of Top equation} \cdot \text{Bottom equation} - \text{Derivative of Bottom equation} \cdot \text{Top equation}}{(\text{Bottom equation})^2} \)

\[ y = \frac{f(x)}{g(x)} \quad \frac{dy}{dx} = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2} \]

2-4 Chain Rule

3 STEPS:  1) Power in front \hspace{0.5cm} 2) Lower power by 1 \hspace{0.5cm} 3) Multiply by derivative of inside

OR  Derivative of outside function · Derivative of inside function

\[ y = (f(x))^n \quad \text{OR} \quad y = f(g(x)) \quad \text{OR} \quad y = f(4x) \quad \text{OR} \quad y = f(x^2) \]

\[ y' = n(f(x))^{n-1} \cdot f'(x) \quad y' = f'(g(x)) \cdot g'(x) \quad y' = f'(4x) \cdot 4 \quad y' = f'(x^2) \cdot 2x \]

EX #1:  \( f(x) = x^2 \tan x \)

EX #2:  \( y = (x^3 - 5)^6 \)

EX #3:  \( f(x) = \frac{\sin x}{\cos x} \)

EX #4:  \( y = 2x^4 \sqrt{x^2 - 5} \)
2-5 Implicit Differentiation

The differentiable functions we have encountered so far can be described by equations in which "y" is expressed in terms of "x". We can also find the derivative of the equation expressed in terms of x and y.

*Implicit Differentiation: function in terms of x's and y's

\[
\left( \text{must write } \frac{dy}{dx} \text{ everytime you take a deriv. of } y \right)
\]

**EX:** \[x^2 - xy + 3y^2 = 7\]

\[
2x - \left[ y(1) + \frac{dy}{dx} \right] + 6y \frac{dy}{dx} = 0
\]

\[
2x - y - x \frac{dy}{dx} + 6y \frac{dy}{dx} = 0
\]

\[
-x \frac{dy}{dx} + 6y \frac{dy}{dx} = -2x + y
\]

\[
\frac{dy}{dx}(-x + 6y) = -2x + y
\]

\[
\frac{dy}{dx} = \frac{-2x + y}{-x + 6y}
\]

**EX#1:** \[x^2 + y^2 = 16 \quad (3, \sqrt{7})\]

Find the equation of the tangent line at the given point.

**EX#2:** Find first two derivatives of \[x^3y^2 = 5\]

**EX#3:** Find \[\frac{dy}{dx} \] \[5x^2 - xy^2 + 3y = x\]

**EX#4:** Find \[\frac{dy}{dx} \] \[5x^2 - xy^2 + 3y = x\]

(Use shortcut)
Questions from CH.2 TEST

Derivative of a picture

\[ f''(1) = \quad f''(3) = \quad f''(4) = \]
\[ f'(6) = \quad f'(7) = \quad f'(8) = \]

Equation of the tangent line at \( x = 1 \)

Equation of the tangent line at \( x = 4 \)

Derivative of a chart

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>58</td>
<td>63</td>
<td>72</td>
<td>61</td>
<td>62</td>
<td>69</td>
<td>61</td>
<td>74</td>
<td>67</td>
</tr>
</tbody>
</table>

EX: \[ f'(40) = \frac{61 - 63}{60 - 20} = \frac{-2}{40} = \frac{-1}{20} \]
EX: \[ f'(130) = \frac{74 - 61}{140 - 120} = \frac{13}{20} \]

EX#1: \[ f'(140) = \]
EX#2: \[ f'(110) = \]

How to read Derivative by Definition Problems

This problem means take a derivative of \( \cos x \).

EX: \[ \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = -\sin x \]

This problem means take a derivative of \( 10x^3 \) at \( x = 2 \).

EX: \[ \lim_{h \to 0} \frac{10(2+h)^3 - 80}{h} = 120 \]

Solve each

EX#1: \[ \lim_{h \to 0} \frac{5(x+h)^4 - 5x^4}{h} = \]
EX#2: \[ \lim_{h \to 0} \frac{(3+h)^4 - 81}{h} = \]

EX#3: \[ \lim_{h \to 0} \frac{\tan(x+h) - \tan x}{h} = \]
EX#4: \[ \lim_{h \to 0} \frac{\sin\left(\frac{\pi}{6} + h\right) - \frac{1}{2}}{h} = \]
2.6 Related Rates

We take derivatives with respect to $t$ which allows us to find velocity. Here is how you take a derivative with respect to $t$:

derivative of $x$ is $\frac{dx}{dt}$, derivative of $y^2$ is $2y\frac{dy}{dt}$, derivative of $r^3$ is $3r^2\frac{dr}{dt}$, derivative of $t^2$ is $2t\frac{dt}{dt} = 2t$

$V$ means volume; $\frac{dV}{dt}$ means rate of change of volume (how fast the volume is changing)

$r$ means radius; $\frac{dr}{dt}$ means rate of change of radius (how fast the radius is changing)

$\frac{dx}{dt}$ is how fast $x$ is changing; $\frac{dy}{dt}$ is how fast $y$ is changing

<table>
<thead>
<tr>
<th>Volume of a sphere</th>
<th>Surface Area of a sphere</th>
<th>Area of a circle</th>
<th>Circumference of a circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = \frac{4}{3}\pi r^3$</td>
<td>$A = 4\pi r^2$</td>
<td>$A = \pi r^2$</td>
<td>$C = 2\pi r$</td>
</tr>
<tr>
<td>$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$</td>
<td>$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$</td>
<td>$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$</td>
<td>$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$</td>
</tr>
</tbody>
</table>

Volume of a cylinder

$V = \pi r^2 h$

$r$ is not a variable in a cylinder because its' value is always the same

Volume of a cone

$V = \frac{1}{3} \pi r^2 h$  
use $\frac{r}{h} = \text{constant}$

Due to similar triangles, the ratio of the radius to the height is always the same. Replace $r$ or $h$ depending on what you are looking for.

**EX #1:** Suppose a spherical balloon is inflated at the rate of 14 in$^3$/min. How fast is the radius of the balloon changing when the radius is 7 inches?

**EX #2:** Water is poured into a cylinder with radius 5 at the rate of 11 in$^3$/s. How fast is the height of the water changing when the height is 4 inches?
EX #3: Water is leaking out of a cone with diameter 10 inches and height 9 inches at the rate of 5 in³/s. How fast is the radius of the water changing when the radius is 4 in?

EX #4: A 17 foot ladder is leaning against the wall of a house. The base of the ladder is pulled away at 3 ft. per second.

a) How fast is the ladder sliding down the wall when the base of the ladder is 15 ft. from the wall?

b) How fast is the area of the triangle formed changing at this time?

c) How fast is the angle between the bottom of the ladder and the floor changing at this time?

EX #5: A person 6 ft. tall walks directly away from a streetlight that is 13 feet above the ground. The person is walking away from the light at a constant rate of 4 feet per second.

a) At what rate, in feet per second, is the length of the shadow changing?

b) At what rate, in feet per second, is the tip of the shadow changing?
7-7 Indeterminate Forms and L'Hôpital's Rule

L'Hôpital's Rule: If \( \lim_{x \to a} \frac{f(x)}{g(x)} = 0 \) or \( \pm \infty \), then \( \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \)

**EX#1:** \( \lim_{x \to 2} \frac{x^2 + 3x - 10}{x - 2} = \)

**EX#2:** \( \lim_{x \to 0} \frac{\sin x}{x} = \)

**EX#3:** \( \lim_{x \to 0} \frac{e^x}{x^3} = \)

**EX#4:** \( \lim_{x \to \infty} \frac{x^2}{e^x - 1} = \)

**EX#5:** \( \lim_{x \to 0} \frac{3e^x - 3x - 3}{x^2} = \)
Exponential and Logarithmic Derivatives Worksheet

This worksheet is arranged in order of increasing difficulty.

For problems 1-8, find the derivative of the given function:

1. \( f(x) = \ln(x) \)
2. \( f(x) = e^x \)
3. \( f(x) = 2^x \)
4. \( f(x) = \log_{10}(x) \)
5. \( f(x) = 8^x \log_6(x) \)
6. \( f(x) = \log_4(x) + 16^x \)
7. \( f(x) = 4e^x \ 4^x \)
8. \( f(x) = 6\ln(x) \)

For problems 9-13, find the derivative of the function at the given point:

9. \( f(x) = 2e^x \quad x, \quad \text{at } x = 1 \)
10. \( f(x) = x^3 + 5x, \quad \text{at } x = 2 \)
11. \( f(x) = \ln(x) \ 3^x, \quad \text{at } x = 3 \)
12. \( f(x) = 6 \ 5^x + \log_{10}(x), \quad \text{at } x = 2 \)
13. \( f(x) = 10 \ e^x + 7x, \quad \text{at } x = 0 \)

For problems 14-28, find the derivative of the given function:

14. \( f(x) = e^{3x} \)
15. \( f(x) = e^{3x^2} \)
16. \( f(x) = \frac{5x}{e^x} \)
17. \( f(x) = \frac{3x^3}{e^x} \)
18. \( f(x) = x^3 \ln(x) \)
19. \( f(x) = \log_2(3x) \)
20. \( f(x) = \log_5(x^2 + 1) \)
21. \( f(x) = \frac{\log_{10}(x)}{x} \)
22. \( f(x) = \frac{e^{2x}}{x} \)
23. \( f(x) = \frac{(e^x)^4}{x^2} \)
24. \( f(x) = x^2 \ln(x^2 + 3x) \)
25. \( f(x) = x^3 \ 8^x \)
26. \( f(x) = \frac{(2x)^2}{e^{2x}} \)
27. \( f(x) = x^2 \log_5(x^2) \)
28. \( f(x) = \frac{e^{2x}}{x^2} \)