Welcome to Math Analysis 2 Honors!

PLEASE READ THE FOLLOWING IN ITS ENTIRETY
I hope you have enjoyed your summer break so far, but now it is time to get back into “school-mode” and review some topics before starting MA2. We start MA2 with Matrices, Combinatorics, Probability, & Statistics so it is very important that these topics are refreshed before you return in September.

Before you begin Math Analysis 2, you will be expected to have mastered the following topics:
• Matrix Addition, Subtraction, and Scalar Multiplication.
• How to identify Matrix dimensions and Matrix Transposition.
• The Multiplication, Addition, and Complement Principle of Counting. Mutually Exclusive
• Probability of a Single event.
• Probability of either of two events and probability of either of two mutually exclusive events.
• Probability of events occurring together.
• How to find Mean, Median, Mode, Range, Upper Extreme, Lower Extreme, Interquartile Range, Quartile 1, and Quartile 3.
• Box-and-Whisker Plots, Stem-and-Leaf Plots, and Histograms.

Login with your Bergen Email and use the MA2 textbook as needed here.

**THERE WILL BE A TEST THE FIRST FEW DAYS OF SCHOOL ON THIS MATERIAL**

Supplies needed for MA2:
1. Pencils for graded assignments.
2. TI-84 (or higher).
4. An iPad stylus (Suggested models: here or here).

Math Analysis 2 is currently a paperless course. If you have your own tablet device and would like to complete this summer assignment on that device, please do so. During the year we will be using iPads, the iOS application Notability, and a stylus (see number 4 above) to take notes.

This year will be challenging and a lot of work for you. But if you have a good attitude and work ethic it can also be a fun and rewarding course to set you on course for the AP Calculus track here at Bergen Tech. Email me if you have any questions. See you soon.

Mr. Mellina (nicmel@bergen.org)
Matrices (Chapter 14)

**Example 1**

Given: $A = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, B = \begin{bmatrix} 2 & 5 \end{bmatrix}, C = \begin{bmatrix} 5 & 2 \end{bmatrix}, D = \begin{bmatrix} 2 & \frac{10}{2} \end{bmatrix}$

a. Give the dimensions of each matrix  

b. Which matrices are transposes of each other?

c. Which matrices are equal to each other?

**Example 2:**

Let $A = \begin{bmatrix} 3 & 8 & 1 \\ 4 & 0 & -3 \\ -2 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 9 \\ 4 & -6 & -5 \\ 0 & 7 & 2 \end{bmatrix}$, find

a. $A + B$  
b. $A - 2B$

c. $A + B^t$
Example 3:
The Handy Hardware Company has two locations, one downtown and one at the mall. During April, the downtown store sold 31 of the lowest-priced lawn mowers, 42 of the medium-priced ones and 18 of the highest-priced mowers. Also during April, the mall store sold 22 of the lowest-priced mowers, 25 of the medium-priced mowers, and 11 of the highest-priced mowers.

a. Represent this information in an April sales matrix $A$.

b. Do the dimensions of your matrix $A$ represent a “mower-type by location” matrix or a “location by mower-type” matrix?

c. Suppose that during May, the Handy downtown store sold 28 of the lowest-priced mowers, 29 of the medium-priced ones and 20 of the highest-priced ones, and that the mall store sold 20 lowest-priced ones, 28 medium-priced ones, and 9-highest priced ones. Represent this information in a May Matrix $M$ that has the same dimensions as $A$.

d. Find $A + M$ and describe what this matrix sum tells you.

e. If the manager of the Handy stores expects next year’s lawn mower sales to rise about 8%, about how many highest-priced mowers does the manager expect to sell at the downtown store next April? If you were the manager, would you round your calculations up or down? What scalar multiple of matrix $A$ would assist you in planning next April’s sales?
Example 4:
Simplify

a. \[ \begin{bmatrix} -3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \]  
b. \[ \begin{bmatrix} 8 & 1 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} \]

c. \[ \begin{bmatrix} 8 & 2 & -2 \\ -3 & 1 & 14 \end{bmatrix} + \begin{bmatrix} 12 & 3 & 10 \\ 0 & 0 & -6 \end{bmatrix} \]  
d. \[ 8 \begin{bmatrix} 5 & -2 \end{bmatrix} \]

e. \[ 2 \begin{bmatrix} 3 & 0 \\ -4 & 1 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} -2 & -2 \\ 3 & 0 \\ 6 & 11 \end{bmatrix} \]  
f. \[ \begin{bmatrix} 1 & -4 & 2 \\ 5 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & -2 \end{bmatrix} \]

Example 5:
Let \( A = \begin{bmatrix} 3 & 2 & 4 & 8 \\ 0 & 1 & 8 & -2 \\ 3 & 7 & 9 & 1 \end{bmatrix} \), \( B = \begin{bmatrix} 8 & 1 & 4 \\ 3 & 0 & 7 \\ -2 & 5 & 11 \\ 6 & 4 & -3 \end{bmatrix} \), \( C = \begin{bmatrix} 9 & 0 & 5 \\ 4 & 3 & 1 \\ 7 & 12 & 2 \\ 6 & 0 & 3 \end{bmatrix} \), find

a. \( A^t \)  
b. \( A^t + B \)  
c. \( 3C - A^t \)
Example 6:
Find the values of the variable for which the given statement is true

a. \[
\begin{bmatrix}
2x - 3y & 3 \\
-7 & 24 - y
\end{bmatrix}
= \begin{bmatrix}
-2 & w \\
-7 & 4x
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
y & 5 \\
2 - c & 3x + \frac{y}{3}
\end{bmatrix}
= \begin{bmatrix}
36 - 9x & 2a + 1 \\
-8 & 12
\end{bmatrix}
\]
Define the following:

**Multiplication Principle of Counting:**

**Mutually Exclusive Events:**

**Addition Principle of Counting:**

**Complement Principle of Counting:**

**Students must familiarize themselves with the contents of a standard 52-deck of cards**

Example 7:
Paula is going to choose the size, color, phrase, and picture for a birthday card for her friend. There are 22 sizes, 44 colors, 77 phrases, and 44 pictures for her to choose from. (The printing company charges a fee to add extra design elements, so she will choose only one of each.) How many different card designs are possible?

Example 8:
Tony allows his customers to create their own pizza. The customers choose exactly one type of crust, sauce, cheese, and meat for their pizza. The following table shows the options available to customers. How many different pizza combinations are there?

<table>
<thead>
<tr>
<th>Crust options</th>
<th>Thin, Traditional, Deep dish, Stuffed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sauce options</td>
<td>Tomato, Pesto, Alfredo, Ranch</td>
</tr>
<tr>
<td>Cheese options</td>
<td>Mozzarella, Cheddar, Italian blend</td>
</tr>
<tr>
<td>Meat options</td>
<td>Pepperoni, Sausage, Chicken, Bacon</td>
</tr>
</tbody>
</table>

Example 9:
In how many ways can 8 people line up in a cafeteria line?
Example 10:
How many license plates can be made using 2 letters followed by 3 digits?

Example 11:
Determine whether the following events are mutually exclusive or not.

a. Turning Left and Turning Right
b. Tossing a coin and getting heads and tossing a coin and getting tails.
c. Turning left and scratching your head
d. In a single draw, pulling a King or pulling an Ace.
e. In a single draw, pulling a King or pulling a Heart.

f. A pair of dice is rolled. Rolling a 9 and rolling a double.
g. A pair of dice is rolled. Rolling a 6 and rolling a double.

Example 12:
A high school coach must decide on the batting order for a baseball team of 9 players.

a. The coach has how many different batting orders from which to choose?

b. How many different batting orders are possible if the pitcher bats last?


c. How many different batting orders are possible if the pitcher bats last and the team’s best hitter bats third?
Example 13:
How many ways can 2 Non-Fiction, 3 Fiction, and 2 biographies books be put on a shelf if they have to be next to their specified genres?

Example 14:

a. Find the number of 4-digit numbers containing at least one digit 5.

b. How many 3-digit numbers contain no 7’s?

c. How many 3-digit numbers contain at least one 7?

d. How many numbers from 5000 to 6999 contain at least one 3?

Example 15:

a. In how many different orders can you arrange 5 books on a shelf?

b. In how many different ways can you answer 10 true-false questions?

c. In how many ways can 4 people be seated in a row of 12 chairs?

d. In how many ways can 4 people be seated in a row of 12 chairs if they must all sit next to each other?
Probability (Chapter 16)

Define the following:

Empirical Probability:

Theoretical Probability:

If two events A and B, then \( P(A \text{ or } B) = \)

If two events A and B are mutually exclusive, then \( P(A \text{ or } B) = \)

**Students must familiarize themselves with the contents of a standard 52-deck of cards**

Example 16:
You toss a dime twice. Copy and complete the table provided to show all the possible outcomes. You can get “heads” or “tails” on each toss.

<table>
<thead>
<tr>
<th>First Toss</th>
<th>Second Toss</th>
</tr>
</thead>
</table>

a. Find the probability that you get the same outcome on both tosses?

b. Find the probability that you get at least one “heads.”

c. Find the probability that you get “heads” on the first toss.

Example 17:
Suppose a die is rolled.

a. Give a sample space for this experiment. Then find the probability of rolling a prime number.

b. Then find the probability of a number rolled by a die will be divisible by 2 or 3.

c. Find the probability of rolling a 3 or rolling an even number.
Example 18:
If a standard deck of 52 cards is well shuffled, what is the probability that the top card is:

a. a black ace?  
   b. not a black ace?  
   c. a diamond face card

Example 19:
Suppose a red die and a white die are rolled. What is the probability that the sum of the numbers showing on the dice is 9 or 10?

Example 20:
Suppose a die is rolled. Find each probability.

a. $P(\text{perfect square})$  
   b. $P(\text{factor of 60})$  
   c. $P(\text{negative number})$

Example 21:
If a card is drawn at random from a standard deck of 52 cards, what is the probability of getting:

a. The queen of hearts  
   b. A heart?  
   c. A queen?

d. A red card?  
   e. A face card?  
   f. A red face card?

g. A red diamond?  
   h. a jack or king?  
   i. Not a black diamond?
Example 22:
Mr. and Mrs. Smith each bought 10 raffle tickets. Each of their three children bought 4 tickets. If 4280 tickets were sold in all, what is the probability that the grand prize winner is:

a. Mr. or Mrs. Smith? b. One of the 5 Smith’s? c. None of the Smith’s

Example 23:
A die is rolled and a coin is tossed.

a. Make a tree diagram showing the 12 possible outcomes of this experiment. b. Find the probability that the die’s number is even and the coin is “heads.”

Example 24:
Suppose you roll two dice, each of which is a regular octahedron with faces numbered 1 to 8.

a. What is the probability that the sum of the numbers showing is 2?

b. What is the probability that the sum is 3?

c. What sum is most likely to appear?
Example 25:

a. Find the mean of the following numbers: 2, 37, 40, 44, 45.

b. Does the mean found in part (a) represent the data well? Explain your answer.

c. Is the mean of a group of numbers always, sometimes, or never a number in the group?

Example 26

The 12 National League baseball teams had shutouts for one season as shown in the table below.

<table>
<thead>
<tr>
<th>Team</th>
<th>Shutouts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta</td>
<td>4</td>
</tr>
<tr>
<td>Chicago</td>
<td>10</td>
</tr>
<tr>
<td>Cincinnati</td>
<td>13</td>
</tr>
<tr>
<td>Houston</td>
<td>15</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>24</td>
</tr>
<tr>
<td>Montreal</td>
<td>12</td>
</tr>
<tr>
<td>New York</td>
<td>22</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>6</td>
</tr>
<tr>
<td>Pittsburgh</td>
<td>11</td>
</tr>
<tr>
<td>St. Louis</td>
<td>14</td>
</tr>
<tr>
<td>San Diego</td>
<td>9</td>
</tr>
<tr>
<td>San Francisco</td>
<td>13</td>
</tr>
</tbody>
</table>

a. Summarize the data in a stem-and-leaf plot.

b. Find the mean, median, and mode.
Example 27
The table below gives the results of a driver-education experiment. The experiment measures the time between the appearance of a stimulus on a screen and a student’s reaction of depressing a brake pedal.

<table>
<thead>
<tr>
<th>Time (to the nearest 0.1 s)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

a. Draw a histogram for the data.

c. Find the mean, median, and mode.

Example 28
The frequency table below summarizes the numbers of siblings (brothers and sisters) for each of the 25 students in a statistics class. For the data, find the following.

<table>
<thead>
<tr>
<th>Number of siblings</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>2</td>
<td>10</td>
<td>8</td>
<td>4</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

a. mean

b. median

Example 29
a. The number of trials required by 20 different puppies to learn a trick is recorded in the stem-and-leaf plot below. Rearrange the data in increasing order, and then find the median and the mode of the data.

```
0 | 9 7 9 4 8
1 | 8 2 3 0 9 1 6 4 5 3
2 | 1 2 8 5
```

Stem-and-leaf plot with 3 stems

b. The stem-and-leaf plot above has just three stems (0, 1, and, 2). In order to spread out the data more, you can draw a plot with 6 stems, as shown below. The stem 2 is used for data from 20 to 24, while the dot below the 2 is the stem for data from 25 to 29. The other dots have similar meanings. Complete the plot.

```
0 |
1 |
2 |
```

Stem-and-leaf plot with 6 stems
Example 30
The following data are the average low monthly temperatures (°F) for Berlin, Germany:

<table>
<thead>
<tr>
<th>J</th>
<th>F</th>
<th>M</th>
<th>A</th>
<th>M</th>
<th>J</th>
<th>J</th>
<th>A</th>
<th>S</th>
<th>O</th>
<th>N</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>27</td>
<td>32</td>
<td>38</td>
<td>46</td>
<td>51</td>
<td>55</td>
<td>54</td>
<td>48</td>
<td>41</td>
<td>33</td>
<td>29</td>
</tr>
</tbody>
</table>

a. Find the median temperature.

b. Find the median of those temperatures that are less than the median temperature.

c. Find the median of those temperatures that are greater than the median temperature.

d. Suppose the data were given in degrees Celsius rather than degrees Fahrenheit. What differences would you expect in the answers to Exercises (a-c)?

Example 31
The stem-and-leaf plot shows the ages of the first 41 U.S. Presidents at the time each took office.

<table>
<thead>
<tr>
<th>4</th>
<th>·</th>
<th>5</th>
<th>·</th>
<th>6</th>
<th>·</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>6</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Find the extremes and the range.

b. Find the median and lower and upper quartiles.

c. Are there any outliers?

d. Draw a box-and-whisker plot.
Example 32:
Use the box-and-whisker plot show below. It gives the weights in pounds of the 42 players on a high school football team.

a. What are the extremes? b. What is the range?

c. What are the lower and upper quartiles? d. What is the interquartile range?

e. What is the median?

f. Explain why the weight of 225 lb is an outlier. What are the other outliers?

Example 33:
Compare the two histograms.